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PROBABILITY OF KILL FOR VLA ASROC TORPEDO LAUNCH

by

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March 2009

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The purpose of this thesis is to generate a tactical decision aid (TDA) capable of calculating the probability of kill of a submarine when targeted with a vertical launched (VLA) anti-submarine rocket propelled torpedo (ASROC). In determining the submarine specific probability of kill (Pk), the Passive Contact Tracker and Kill Probability (PACT-AKP) TDA will calculate the submarine's position and its area of uncertainty (AOU) based on single or multiple ASW passive sensor bearing-to-target inputs.

In determining the target's position and AOU, PACT-AKP employs an extended Kalman Filter that uses MTST movement and measurement models. In calculating ASROC probability of kill, submarine specific torpedo specific effectiveness (TEFF) data collected from NUWC Newport was used to generate the Pk algorithm.

We can conclude that PACT-AKP not only assists the ASW team with target motion analysis (TMA), but also provides the commander with a credible target probability of kill prior to the employment of VLA ASROC torpedoes as a deliberate attack weapon.

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PROBABILITY OF KILL FOR VLA ASROC TORPEDO LAUNCH

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ABSTRACT

The purpose of this thesis is to generate a tactical decision aid (TDA) capable of calculating the probability of kill of a submarine when targeted with a vertical launched (VLA) anti-submarine rocket propelled torpedo (ASROC). In determining the submarine specific probability of kill (Pk), the Passive Contact Tracker and Kill Probability (PACT-AKP) TDA calculates the submarine's position and its area of uncertainty (AOU) based on single or multiple ASW passive sensor bearing-to-target inputs.

In determining the target's position and AOU, PACT-AKP employs an extended Kalman Filter that uses MTST movement and measurement models. In calculating ASROC probability of kill, submarine specific torpedo specific effectiveness (TEFF) data collected from NUWC Newport was used to generate the Pk algorithm.

We can conclude that PACT-AKP not only assists the ASW team with target motion analysis (TMA), but also provides the commander with a credible target probability of kill prior to the employment of VLA ASROC torpedoes as a deliberate attack weapon.

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TABLE OF CONTENTS

I. PACT-AKP **1**
- **A. INTRODUCTION** **1**
- **B. BACKGROUND** **1**
- **C. PURPOSE** **2**
- **D. SCOPE AND METHODOLOGY** **3**
- **E. BENEFITS OF PACT-AKP** **3**

II. PACT-AKP METHODOLOGY **5**
- **A. INTRODUCTION** **5**
- **B. METHODOLOGY** **5**
- **C. EXTENDED KALMAN FILTER** **5**
  - **1. Maneuvering Target Statistical Tracker (MTST)** **6**
    - ***a. PACT-AKP MTST Movement Model*** ***7***
    - ***b. PACT-AKP MTST Measurement Model*** ***8***
    - ***c. PACT-AKP Initial Target Position Guess Entry*** ***9***
    - ***d. PACT-AKP Coordinate System*** ***10***
    - ***e. MTST Shock and Dimensionless Shock*** ***11***
- **D. PROBABILITY OF KILL CALCULATIONS** **11**
  - **1. Target Relative to Weapon Water Entry Point AOU** **11**
  - **2. Torpedo Effectiveness (TEFF)** **13**
  - **3. Probability of Kill Calculation** **14**
    - ***a. Derivation and Assumptions of PACT-AKP Probability of Kill Algorithm*** ***15***
    - ***b. Probability of Kill Algorithm*** ***16***

III. PACT-AKP PERFORMANCE TESTING **19**
- **A. SCOPE** **19**
- **B. PACT-AKP MEASUREMENT MODEL PERFORMANCE TEST** **19**
  - **1. Target Position Guess Reliability Test** **19**
  - **2. Single Sensor Measurement Model Performance Test** **20**
    - ***a. PACT-AKP Measurement Model Single Sensor and Single Bearing-to-target AOU*** ***20***
    - ***b. PACT-AKP Measurement Model Single Bearing-to-target Position Test*** ***23***
  - **3. Multiple Sensor Measurement Model Performance Test** **23**
    - ***a. Two Sensor Measurement Model Test*** ***24***
- **C. PACT-AKP MOVEMENT MODEL PERFORMANCE TEST** **25**
  - **1. Change in Time Movement Model Test** **25**
- **D. PACT-AKP EKF PERFORMANCE TEST** **25**
  - **1. Composite EKF Performance Test** **26**
  - **2. Dimensionless Shock (DS) Test** **35**
- **E. PACT-AKP PROBABILITY OF KILL ALGORITHM TEST** **37**

IV. CONCLUSION 41
- **A. INTRODUCTION.....41**
- **B. RECOMMENDATIONS FOR FUTURE AREAS OF STUDY 41**
- **C. CONCLUSIONS 42**

LIST OF REFERENCES 43

INITIAL DISTRIBUTION LIST 45

LIST OF FIGURES

Figure 1. Single Station Measure the Bearing to a Target Located at (X, Y). 8
Figure 2. Bearing-to-target Measurement vs. Expected Bearing-to-target. 11
Figure 3. TEFF Data Display. 14
Figure 4. Target Position Guess AOU Display. 20
Figure 5. North-South Single Sensor Bearing-to-target Target AOU. 21
Figure 6. East-West Single Sensor Bearing-to-target Target AOU. 22
Figure 7. Two Sensor Measurement Target AOU. 24
Figure 8. Case One Scenario at Time 0 After the First Two Measurements. 28
Figure 9. Case Two Scenario at Time 0 After the First Two Measurements. 28
Figure 10. Case Three Scenario at Time 0 After the First Two Measurements. 29
Figure 11. Case One Target Track at Time 1 After Four Bearing Measurements. 30
Figure 12. Case Two Scenario at Time 1 After Four Bearing Measurements. 30
Figure 13. Case Three Scenario at Time 1 After Four Measurements. 31
Figure 14. Case One Scenario at Time 2 After Six Measurements. 31
Figure 15. Case Two Scenario at Time 2 After Six Measurements. 32
Figure 16. Case Three Scenario at Time 2 After Six Measurements. 33
Figure 17. Case One Scenario at Time 3 After Six Measurements. 33
Figure 18. Case Two Scenario at Time 3 After Six Measurements. 34
Figure 19. Case Three Scenario at Time 3 After Six Measurements. 35
Figure 20. Pk vs. *S1*. 40

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LIST OF TABLES

Table 1. North-South Single Sensor Bearing-to-target Measurement Station Data. 20
Table 2. East-West Single Sensor Bearing-to-target Measurement Station Data. 22
Table 3. Target Position Test Measuring Station Data. 23
Table 4. Multiple Sensors Measuring Station Data. 24
Table 5. Change in Time 2-Sigma AOU Expansion...25
Table 6. Actual Target Track Data..26
Table 7. Measurement Station Data (Heterogeneous Sensor Error). 27
Table 8. Measurement Dimensionless Shocks...36
Table 9. Case Two Scenario Target Relative to Weapon Water Entry Point AOU Data. 37
Table 10. Converted TEFF Data. 37
Table 11. Target relative to Torpedo Water Entry Point AOU Data. 38
Table 12. Pk Results...38
Table 13. Circular 2-Sigma AOU Probability of Kill Results. 39

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EXECUTIVE SUMMARY

With limited weapon payload and increasing number of potential threat submarines, if the U.S. surface fleet is to contribute to maintaining maritime dominance the submarine threat must be met with the prudent and effective use of anti-submarine warfare (ASW) munitions. This thesis explores and solves a current and tactically critical ASW problem that plagues the surface navy fleet; the probability of kill of a submarine when engaged with a vertical launched (VLA) anti-submarine rocket propelled light weight torpedo (ASROC). This thesis develops the passive contact tracker and probability of kill (PACT-AKP), an EXCEL-based Microsoft Windows platform tactical decision aid (TDA) designed to aid in the passive tracking of a target and more importantly, to determine the target's probability of kill when engaged with a VLA ASROC.

PACT-AKP only requires the user to enter passive contact information normally recorded and processed during passive target motion analysis (TMA). Upon demand, PACT-AKP provides a graphical depiction of the submarine's position, 2-sigma area of uncertainty (AOU) and ASROC probability of kill. In addition, PACT-AKP's probability of kill feature is flexible enough so that it gives the user the ability to calculate the probability of kill without using the program's passive contact tracker feature.

Through use of Kalman filter theory and probability theory, this study provides a probability of kill for ASROC employment. Although numerous naval systems employ Kalman filters and its variations, what makes this thesis unique is the derivation of the probability of kill algorithm. Prior to this effort, there was no antecedent work tackling this problem. Anti-submarine warfare operations are complex and beset with uncertainty. Determining a submarine's probability of kill when engaged with a VLA ASROC prior to the actual deliberate attack engagement provides the commander with the necessary knowledge needed to make better decisions concerning the expenditure of limited

shipboard ASW munitions. The probability of kill algorithm developed herein is unique and provides a foundation for the development of future Navy-wide ASW doctrine governing the use of ASROC munitions.

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I. PACT-AKP

A. INTRODUCTION

With limited weapon payload and increasing number of submarines world wide, if the U.S. surface fleet is to continue to maintain maritime dominance the ASW threat must be met with the prudent and effective use of ASW munitions.

This thesis explores and answers a current ASW problem facing the surface fleet, estimation of the probability of kill of a submarine when engaged with a vertical launched (VLA) anti-submarine rocket propelled light weight torpedo (ASROC).

This thesis develops the passive contact tracker and probability of kill (PACT-AKP), a Microsoft Windows platform tactical decision aid (TDA) designed to aid in the passive tracking of a target as well as to determine the target's probability of kill when engaged with a VLA ASROC.

B. BACKGROUND

When faced with an ASW threat, both the commander and the ASW team should be equipped with readily available decision aids capable of providing valuable and practical information. Passive tracking of a submarine using target motion analysis (TMA) is normally conducted by hand, however, this tedious method can be easily replaced or the process aided by a TDA. Moreover, the commander should also have the necessary information available to make an informed decision before engaging in the expenditure of the warship's limited ASW munitions to neutralize the ASW threat.

Because current inventories of VLA MK46 light weight torpedoes are low, determining the probability of kill for a hostile submarine is important. As defined by NTTP-3-21.33 Surface Torpedo Attack Tactics / Countermeasures / Evasion Manual, a deliberate attack is offensive in nature and it is a planned coordinated employment of firepower intended to destroy a hostile submarine. The most effective ASW weapon for the prevailing circumstance should be used. Due to the short employment ranges of

surface vessel torpedo tube (SVTT) launched torpedoes that requires maneuvering well within the lethality range of submarine torpedoes, VLAs should be used for deliberate attacks. Surface vessels equipped with MK-41 vertical launch systems and armed with ASROC torpedoes are the preferred surface platform for a deliberate attack due to their standoff weapon engagement ability.

The surface fleet is challenged to realistically determine the probability of killing a submarine when engaged with a VLA ASROC. Despite popular belief, the “Pk” display on the OJ-452 console does not refer to probability of kill, but rather how well the underwater fire control system (UFCS) of the ANISQQ-89 integrated ASW system is tracking a target in terms of up-to-date bearing-to-target measurements. The fleet is currently without a method for assessing kill probability for the actual weapon.

During a major fleet exercise Valiant Shield 2006 (VS06), simulated VLA employment greatly exceeded actual magazine capacity. It was observed that surface escorts followed a policy of “classify with ordinance” or using a weapon in the water to determine if the contact was an actual submarine (Naval Mine and Anti-submarine Warfare Command, (Forthcoming)). VS06 not only identified a weapon misuse issue but also a problem with kill probability determination regarding the VLA ASROC weapon system. The following year, exercise Valiant Shield 2007 (VS07) produced similar results. As published in TM 3-21.1-08 Vertical Launch ASROC (VLA) Employment, escorts expended more simulated torpedoes than physically possible. Based on observations from VS06 and VS07, one of the logical conclusions is that commanders had to know how and when to use ASW weapons more prudently (NMAWC, (Forthcoming)). However, without the ability to ascertain the success of the weapon engagement as measured by the probability of kill, the commander must necessarily make less than optimal weapon expenditure decisions.

C. PURPOSE

This thesis’ purpose is to create a TDA capable of assisting the antisubmarine warfare team with the passive sensor tracking of an ASW threat and assist the

commander in making prudent decisions regarding the employment of the VLA ASROC torpedoes by calculating the probability of kill (Pk).

D. SCOPE AND METHODOLOGY

PACT-AKP requires the user to manually enter passive sensor bearing-to-target data normally used while conducting TMA. PACT-AKP then processes these inputs using an Extended Kalman Filter (EKF) that employs a maneuvering target statistical tracker (MTST) movement and measurement model to estimate the position and velocity of the target. It graphically depicts the target's area of uncertainty (AOU) as derived from the EKF process and calculates its probability of kill when engaged with either a MK-46 or MK-54 ASROC.

PACT-AKP uses unique submarine specific torpedo effectiveness (TEFF) data along with information derived from the EKF process to calculate the probability of kill. PACT-AKP's kill probability algorithm uses TEFF data as determined by the Navy's Weapon Analysis Facility (WAF) and published in TACD&E 00-20, 01-14, 02-11, 03-14 and 06-XX . In this thesis, only unclassified data is used to demonstrate PACT-AKP.

E. BENEFITS OF PACT-AKP

Although the use of Kalman filters for the purpose of TMA is not a novelty, what sets PACT-AKP apart from any other TDA in its class is its fleet-wide accessibility, portability and above all, its unique feature of determining probability of kill.

PACT-AKP enhances the capabilities of the Surface Navy by providing an effective passive ASW tracking process, and provides the commander with invaluable insight regarding VLA ASROC engagement allowing the commander to make more informed tactical decisions.

In addition, PACT-AKP's kill probability algorithm can easily be updated as new weapon effectiveness data is made available and its target tracking capability is scalable with slight modification of its embedded visual basic (VBA) code. PACT-AKP enhances the surface fleet's ability to employ the VLA ASROC weapon system.

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II. PACT-AKP METHODOLOGY

A. INTRODUCTION

The purpose of this chapter is to provide the underlying theory and computational methodology used by PACT-AKP to both passively track and determine the kill probability of a target if engaged with a VLA ASROC torpedo.

Throughout this chapter symbols for vectors and matrices will be bold, scalars will be italic and "←" is the replacement symbol. The superscript t on a matrix means "transpose."

B. METHODOLOGY

PACT-AKP employs a variant of the Kalman filter to process user inputs in order to determine and project the state of the target from time t to time $t + \Delta t$. This variant is the extended Kalman filter (EKF).

A basic Kalman filter is a method for recursively updating an estimate $\boldsymbol{\mu}$ of the state of a system by processing a succession of measurements Z. After each measurement step a new estimate is produced. In PACT-AKP, the measurement Z is bearing-to-target while $\boldsymbol{\mu}$ is an estimate of $\mathbf{X}$, the location of the target in Cartesian coordinates. The associated covariance matrix $\boldsymbol{\Sigma}$ is used in calculating the dimensions of a 2-Sigma Ellipse area of uncertainty (AOU). The dimensions of the semi-major and semi-minor axis of the ellipse, as well as the estimated target position, along with TEFF data, are used to calculate kill probability.

C. EXTENDED KALMAN FILTER

In a basic Kalman filter, the measurement Z is related to $\mathbf{X}$ through the equation $Z = \mathbf{HX} + \mathbf{V}$. Z is the bearing-to-target measurement, $\mathbf{H}$ is the measurement matrix that describes how the bearing measurement depends on the target state and $\mathbf{V}$ is are independent Gaussian errors associated with the sensor. Because the relationship

between the bearing measurement Z and the target state is non-linear, the matrix **H** is obtained by linearizing the non-linear function of Z. The logic is as follows, if $Z = f(\mathbf{X}) + \mathbf{V}$ then the approximation $Z \cong f(\boldsymbol{\mu}) + \mathbf{H}(\mathbf{X} - \boldsymbol{\mu}) + \mathbf{V}$, where $\mathbf{H} = df(\mathbf{X}) / d\mathbf{X} \mid \mathbf{X} = \boldsymbol{\mu}$ is the matrix of the first partial derivatives. The non-linear function of the state variables is approximated by the first order terms of a Taylor series expansion about **μ**. Except for the fact that **H** depends on **μ**, the approximation is a linear function of **X** (Washburn, 2007).

1. Maneuvering Target Statistical Tracker (MTST)

The Maneuvering Target Statistical Tracker was developed by Daniel H. Wagner Associates in the early 1980s and was chosen by the Navy as a Standard Tracker for at-sea targets. Currently, it is used in many tactical data systems, including the Global Command and Control System (GCCS) and Tomahawk Weapons Control System (TWCS). MTST builds upon the principles of Kalman filtering theory and makes the filtering process capable of processing non-linear measurements such as the ones associated with bearing-to-target measurements.

In MTST, the state vector is a 4 x 1 matrix taking the following form:

$$\mathbf{X} = \begin{bmatrix} Long \\ Lat \\ Longvel \\ Latvel \end{bmatrix} \rightarrow \begin{bmatrix} X \\ Y \\ Xvel \\ Yvel \end{bmatrix}$$

The extended Kalman filter used in PACT-AKP employs MTST movement and measurement models. Wagner (1989) explains in detail the mathematical theory behind each type of model. What makes the MTST models different from basic Kalman filtering are the particular formulations of the matrices , **Q** which are derived from the Ornstein-Uhlenbeck (O-U) process. The velocity components of the state vector are assumed to be independent Ornstein-Uhlenbeck processes (Wagner, 1989; Washburn, 2007). Both the movement model and measurement model are presented as applicable to PACT-AKP.

a. PACT-AKP MTST Movement Model

PACT-AKP's MTST movement model has two embedded parameters that match submarine specific operating characteristics, and a user input parameter that represents the time in between measurements (Δt). The embedded parameters are the relaxation time (τ) and the root mean squared (RMS) speed (s). These parameters are the fundamental building blocks of the MTST movement projection model and are embedded in PACT-AKP.

The relaxation time (τ) is the average time a target travels on a particular heading before changing course. PACT-AKP assumes τ = 0.5 hrs. The RMS speed (s) is the average instantaneous speed of the submarine in knots. PACT-AKP assumes s = 7 kts.

To project the estimated state vector forward from time t to time $t + \Delta t$, MTST uses the updates $\boldsymbol{\mu} \leftarrow \boldsymbol{\Phi\mu}$ and $\boldsymbol{\Sigma} \leftarrow \boldsymbol{\Phi\Sigma\Phi^t} + \mathbf{Q}$. The matrices $\boldsymbol{\Phi}$ and $\mathbf{Q}$ are given by:

$$\phi = \begin{bmatrix} 1 & 0 & b_1 & 0 \\ 0 & 1 & 0 & b_1 \\ 0 & 0 & b_2 & 0 \\ 0 & 0 & 0 & b_2 \end{bmatrix} ; \mathbf{Q} = \begin{bmatrix} c_1 & 0 & c_2 & 0 \\ 0 & c_1 & 0 & c_2 \\ c_2 & 0 & c_3 & 0 \\ 0 & c_2 & 0 & c_3 \end{bmatrix}$$

Where:

let $c = .5s^2$, where s^2 has units of nm^2/hrs^2

$b_1 = (1 - b_2)\tau$, where b_1 has units of *hrs*.

$b_2 = \exp^{(-\Delta t/\tau)}$, where b_2 is unitless.

$c_1 = c\left(2\Delta t - (3 - 4b_2 + b_2^2)\tau\right)\tau$, where c_1 has units of nm^2.

$c_2 = \dfrac{cb_1^2}{\tau}$, where c_2 has units of nm^2/hr.

$c_3 = c\ (1 - b_2^2\ )$ where c_3 has units of nm^2/hr^2.

b. PACT-AKP MTST Measurement Model

In the user interface of PACT-AKP, the user is required to enter bearing-to-target observations (Z) and the associated passive sensor error. Z is then modeled as:

$$Z = \mathbf{HX} + \mathbf{V}$$

In PACT-AKP, **H** is a 1 x 4 matrix and the $\mathbf{V}$'s are independent Gaussian vectors with mean zero and a 1 x 1 covariance matrix $\mathbf{R}$. The matrix $\mathbf{R}$ is obtained by squaring the standard deviation of the measurement error of the bearing-to-target observation, as inputted by the user. The user is in all cases required to input the accuracy of each bearing, along with the bearing itself.

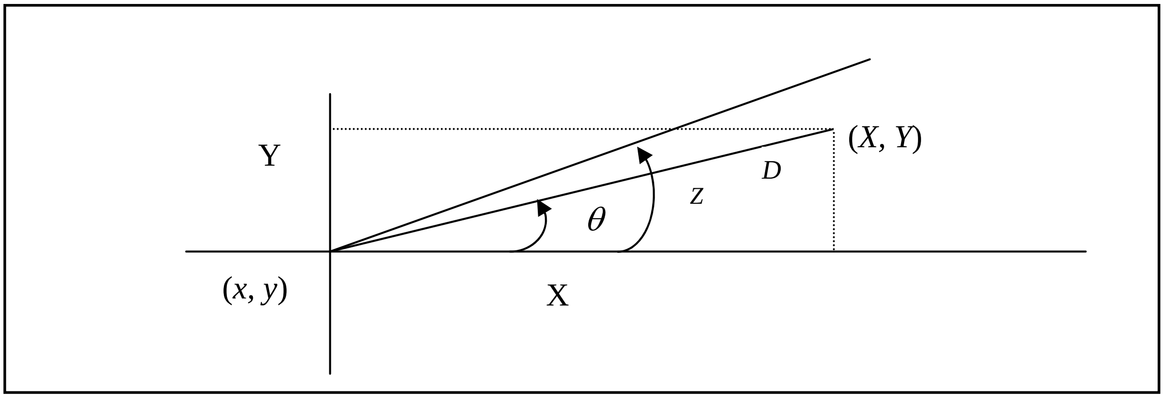


Figure 1. Single Station Measure the Bearing to a Target Located at (X, Y).[1]

Figure 1 provides a visual basis for the derivation of **H**. The location of the target is $(X,Y)^t$, with the observers (measuring stations / ships) located at known points (x,y). Let (D,θ) be the true range and bearing from the observer to the submarine. Consider the measurement $Z = \theta + \mathbf{V}$, where $\theta = \arctan\left(\frac{(Y - y)}{(X - x)}\right)$. The measurement is a non-linear function of the state. Since:

[1] All Figures and Tables within this thesis were created by the Author.

$\frac{d\theta}{dY} = \frac{\cos(\theta)}{D}$ and $\frac{d\theta}{dX} = \frac{-\sin(\theta)}{D}$, and since θ does not depend on the velocity component of $\mathbf{X}$, $\mathbf{H}$ is $[-\sin(\theta)/D,\ \cos(\theta)]/D,\ 0,\ 0]$

Because both θ and D depend on the unknown state of the target, **H** must be evaluated by inserting the latest estimates of θ and D.

The target state is updated following a well defined sequence of calculations:

1. Calculate the Kalman Gain: $\mathbf{K = \Sigma H^t \left[H \Sigma H^t + R \right]^{-1}}$
2. Calculate $\boldsymbol{\mu}$ (post-observation state of the target): $\boldsymbol{\mu} \leftarrow \boldsymbol{\mu} + \mathbf{K}S$, where $S = Z - \theta,$ and θ is the bearing to the expected target location.
3. Calculate the post-observation covariance matrix: $\mathbf{\Sigma} \leftarrow \left[\mathbf{I - KH}\right]\mathbf{\Sigma}$, where **I** is the identity matrix.

Wagner (1989) and Bailey (1992) further describe the EKF measurement model.

c. PACT-AKP Initial Target Position Guess Entry

PACT-AKP's EKF requires a guess at the target's state in order to get the filtering process started. The initial target position or "position guess" in PACT-AKP is entered by the user. The upper left 2 x 2 corner of the covariance of the target position is hard-coded into the program with the value 6.25 nm^2, which indicates that PACT-AKP assumes the initial target position guess is an "educated guess" and accurate to within approximately 5nm (2Sqrt(6.25)= 5). This setting is PACT-AKP specific because the assumption is that when conducting passive TMA the operator will have a general idea of the bearing and distance of the target from the sensor by considering environmental conditions and sensor performance.

Like the initial Σ, the target position guess corresponds to $\boldsymbol{\mu}$ at time 0. Once the later is inputted or initialized, all subsequent calculations correspond to either the movements or measurements.

d. PACT-AKP Coordinate System

Most of the filtering process that takes place using MTST is performed on a flat earth. PACT-AKP requires the user to enter all inputs concerning measuring station position and target position guess in spherical coordinates of latitude and longitude, PACT-AKP later converts these coordinates into the X and Y coordinate system.

The coordinate system used to describe the position of the measuring station as well as the target is the local East/North (Cartesian) coordinate system, which is defined at the point of the Earth that touches the tangent plane. The local East represents the X axis and the local North the Y axis.

Almost all calculations are performed in the flat earth coordinate system with the exception of the bearing to the expected target location ($\theta_{\exp ected}$), which is calculated using spherical trigonometry. $\theta_{\exp ected}$ is the EKF's best prediction of Z_i based on all history previous to the i^{th} measurement (Washburn, 1982). PACT-AKP calculates $\theta_{\exp ected}$ using spherical trigonometry in order to maintain measurement accuracy. For instance, a one degree difference between two lines of bearing (LOB) to the same target, one LOB calculated in spherical coordinates and the other using flat earth coordinates can easily over a long distance generate a lateral difference of several miles.

e. MTST Shock and Dimensionless Shock

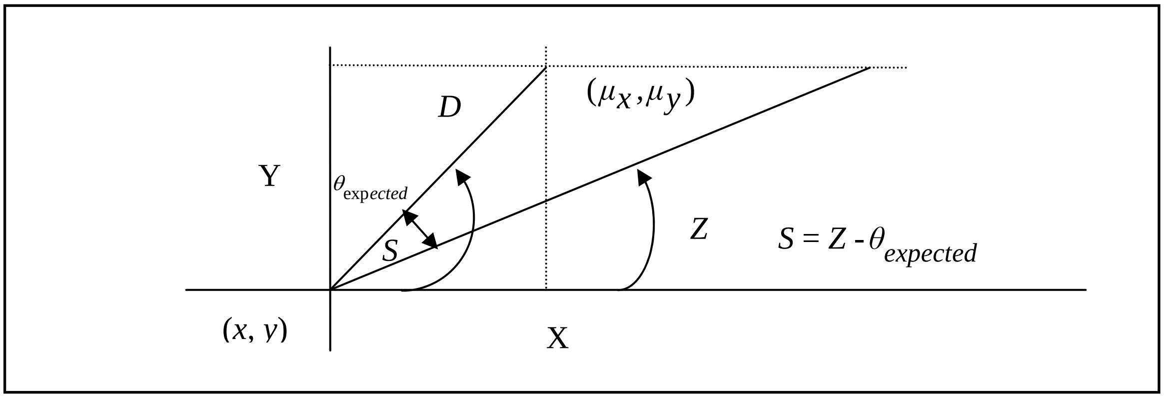


Figure 2. Bearing-to-target Measurement vs. Expected Bearing-to-target.

From Figure 2, the difference between what is actually measured (Z) and $\theta_{expected}$ is called shock (S). From the measurement model $Z = \mathbf{HX} + \mathbf{V}$, the shock is the linear combination of the independent variables **X** and **V**. The approximate expected shock is 0 and the variance of S is $\mathbf{H\Sigma H^t} + \mathbf{R}$, which is the denominator of the Kalman gain computation, and is the means for evaluating when the shock is too large. The dimensionless shock (DS) is given by:

$$DS \equiv S^2 (\mathbf{H\Sigma H^t} + \mathbf{R})^{-1}$$

As Washburn (2007) explains, if the shock has m components, DS should be a scalar random variable that has a Chi-square distribution with m degrees of freedom. When the DS becomes large in comparison to m, it is usually because the filter has lost track. PACT-AKP displays DS after updating the state of the target.

D. PROBABILITY OF KILL CALCULATIONS

1. Target Relative to Weapon Water Entry Point AOU

In calculating the target's AOU, PACT-AKP uses the covariance matrix output from the EKF. The AOU calculated is a 2-Sigma ellipse which is an equiprobability contour that contains the target state with probability equal to 1- exp (-2) or .865

(Washburn, 1989). The ellipse has an inclination *I,* and semi-major axis $2s_1$ and a semi-minor axis $2s_2$. In calculating the dimensions of the ellipse we use the following matrix notation recalling that the output covariance matrix of the EKF is a 4 x 4 matrix consisting of 16 numbers of which we are only interested in using the upper 2 x 2 corner (four numbers).

$$\Sigma = \begin{bmatrix} a & h & & \\ h & b & & \\ & & & \\ & & & \end{bmatrix} \qquad s_1^2 = \left[\left(\frac{a+b}{2} \right) + \sqrt{\left(\frac{a-b}{2} \right)^2 + h^2} \right]$$

$$s_2^2 = \left[\left(\frac{a+b}{2} \right) - \sqrt{\left(\frac{a-b}{2} \right)^2 + h^2} \right]$$

An ASROC will not hit precisely the spot at which it is aimed. The difference between the aim point and the water entry point is assumed to have a circular normal distribution with a circle error probable (ASROC CEP). In calculating the ASROC's fire error (ASROC σ), the ASROC CEP is divided by $\sqrt{2\ln(2)}$ (Washburn, 2002). PACT-AKP combines the ASROC σ with both semi-major and semi-minor axis derived from the EKF covariance matrix as depicted below:

$$S_1 = \sqrt{\left(s_1^2 + ASROC\sigma^2\right)}, S_2 = \sqrt{\left(s_2^2 + ASROC\sigma^2\right)}$$

The resulting semi-major and semi-minor axis are modified to account for the ASROC fire error, thus the target's area of uncertainty becomes relative to the ASROC water entry point. The importance of this transformation becomes evident in the probability of kill calculation by allowing for the use of Washburn's "EllipQ" VBA function. EllipQ builds upon Gilliland's power series derivations for determining the probability of a hit within a specified target-centered circle when it is assumed that the causative missile guidance error is distributed according to the bivariate normal distribution (Gilliland, 1962).

Let R be the location of the target relative to the water entry point of the weapon. R is a critical random variable (RV) for the kill probability calculation and its cumulative density function (CDF) very difficult to calculate except for in the case where $S_1 = S_2$. In this case the CDF of R is:

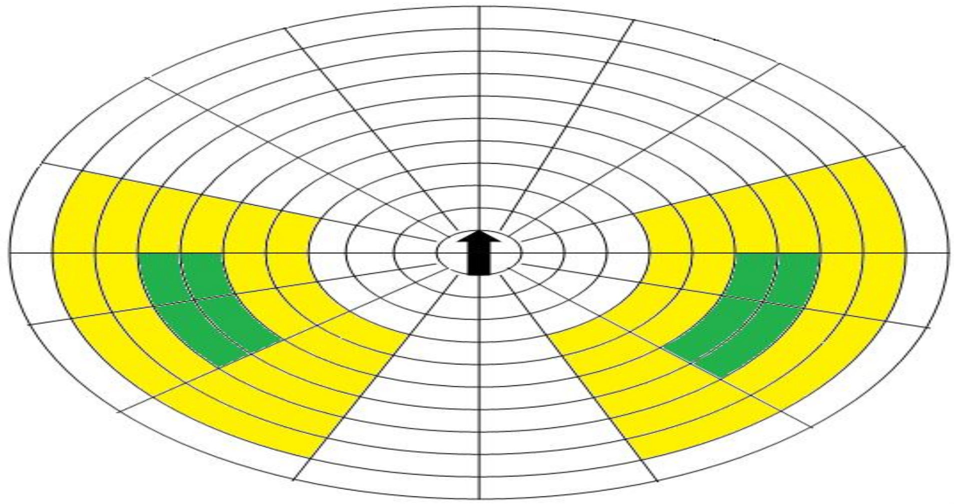
$$\Pr\left(R \leq r\right) = 1 - \exp^{-\left(\frac{r^2}{2S_1^{\,2}}\right)}$$

Unfortunately, the problem at hand does not meet the above condition. The CDF of R as calculated by Washburn's EllipQ function derived from Gilliland's CDF of R derivation power series eq. 13. Gilliland's method can be applied to end-point accuracy problems entailing conditions such as the one explored in this thesis: indefinite target location (Gilliland, 1962). Passive target tracking does not yield an exact target location, but rather a target position contained within a 2-Sigma Ellipse with 86.5% probability. Because of this uncertainty, the indefinite target location condition is applicable and the aforementioned method can be exploited with slight modification so that it is applicable to the problem at hand, the probability of kill of a submarine when engaged with a VLA ASROC.

Washburn's EllipQ function is a fundamental building block in the process of calculating the target's probability of kill. EllipQ requires three inputs. The first input is the lethal radius of the weapon. The second and the third inputs are S_1 and S_2 as calculated for determining the target relative to weapon water entry point AOU.

2. Torpedo Effectiveness (TEFF)

Submarine type, evasion profile, and torpedo type and torpedo running profile specific TEFF data generated by the Weapon Analysis Facility (WAF) was provided by NUWC Newport. Because this information is classified SECRET, data similar in format was used to exercise the PACT-AKP's Pk algorithm. A secret version of PACT-AKP that considers classified TEFF data can be made available. Figure 3 is a visual display of what TEFF data looks like prior to processing and serves as the anchor-point for further TEFF discussions.



Score	Average Teff	Zone (Range/Aspect)
Excellent	60%	Splash-point= 700yds, Asp 113° 600-800yds, Aspect 90° to 136°
Satisfactory	30	400-1000yds, Aspect 68° to 158°
Unsatisfactory	<30%	Everything Else

Figure 3. TEFF Data Display.

As observed from Figure 3, TEFF data is arranged according to angular (aspect) sectors defined by both their angular range and distance from center point. The center point (0, 0) of the graph represents the location of a particular submarine proceeding in the direction of the arrow. The sectors are divided into three categories: excellent, satisfactory, and unsatisfactory. Each sector is assigned a probability score (TEFF) that represents the drop zone torpedo probability of kill against the submarine. The probability of kill is based on which sector relative to the target the torpedo enters the water.

3. Probability of Kill Calculation

This thesis explores a unique, complex and tactically relevant problem that deals with target location uncertainty, weapon system fire error and a weapon system that cannot be described as having a lethal radius but rather a probability of kill as described in the TEFF section.

Although the use of EKFs for tracking targets is not a novelty, PACT-AKP's probability of kill algorithm is the only one of its kind.

a. Derivation and Assumptions of PACT-AKP Probability of Kill Algorithm

From the TEFF data and as depicted in Figure 3, PACT-AKP uses the category sector distances (*r*) from center-point $r_0 = 0$ to the outer radius of the n^{th} TEFF sector; *n*= 1,..,*N*. Where r_n is the outer radius of the n^{th} TEFF sector. Let $Q_n = EllipQ(r_n, S_1, S_2)$.

Let *K* be the event that the ASROC kills the submarine. Figure 3 shows $P(K \mid r, \theta),$ where (r, θ) is the location of target relative to the ASROC water entry point in polar coordinates. In spite of the recommended "splash point," we assume that the ASROC is aimed directly at the submarine (the origin in Figure 3), and that the angle θ is uniformly distributed $[0,\ 2\pi]$. The logic behind these assumptions is as follows:

(1) The submarine's direction is unknown. Despite the best efforts of the TMA process, at the moment of weapon engagement it is plausible that the true direction of the submarine is unknown. The ASW team as well as the underwater fire control system may have a good idea of the general direction of the submarine, but one must not dismiss the possibility that the submarine may have changed course along the way since the last measurement. Because of this uncertainty, we assume that the likelihood of the submarine heading in any particular direction is the same.

(2) The time lapsed between ASROC launch and water entry is minimal. Because of this assumption, leading the target will have little effect and it is assumed that the ASROC is much faster than the submarine.

(3) As depicted in Figure 3, the area comprising the "excellent zone" only accounts for 5.7% of the total TEFF area depicted. The maximum value of the density of the location of the target relative to the ASROC water entry point is given by $\left(\dfrac{1}{2\pi S_1 S_2}\right)$. The probability of the ASROC hitting one of the excellent zones is the product of the area of the excellent zone and the density. Assume that the target relative

to ASROC water entry point 2-Sigma ellipse AOU dimensions are $S_1 = S_2 = 500$ yds , then the probability of hitting one of the excellent zones is only .07, even if one aims at the subject zone.

Since θ is random, the kill probability depends only on r, the distance between the ASROC water entry point and the submarine. Let P_n represent the ring weighted TEFF score; *n*= 1,... *N*.

$$P_n = P(K \mid r) = \frac{1}{2\pi}\int_0^{2\pi} P(K \mid r,\ \theta)d\theta,\ \text{for } r_{n-1} \le r \le r_n$$

In Figure 3 for example, *N*= 4 and $r_1 = 400yds$, $r_2 = 600yds$, $r_3 = 800yds$ and $r_4 = 1000yds$. Also, let P_{N+1} be the kill probability when the miss distance exceeds r_N. The kill probability in the third ring (*n*= 3 and $r_3 =$ 800 yards) and assuming that “< 30%” means 0.15 for the unsatisfactory region is:

$$P_3 = P(K \mid r) = \frac{.6(4) + .3(4) + .15(8)}{16} = .3,\ \text{for } 600 \le r \le 800 yards$$

The above equation represents the weighted TEFF average for ring three defined by $600yrds \le r \le 800yds$. P_{N+1} always corresponds to “unsatisfactory” in TEFF data, and we will always take it to be half of the given bound. Given Figure 3, P_5 would be .15.

b. Probability of Kill Algorithm

PACT-AKP’s probability of kill algorithm bases its computations on the theory of total probability (TTP), as well as the inputs from the weighted sector TEFF and Washburn’s EllipQ function.

In following the previous notation, the Pk algorithm takes as input P_n and Q_n.

$$Pk = \sum_{n=1}^{N} P_n (Q_{n-1} - Q_n) + P_{N+1} Q_N$$

The difference $(Q_{n-1} - Q_n)$ is the probability that the ASROC water entry point is in the n^{th} ring.

Although the algorithm is simple and elegant, the complexity resides in deriving the various elements that comprise the equation.

Consider PACT-AKP probability of kill algorithm and the TEFF data in Figure 3. If one wished to calculate the Pk for the submarine represented in Figure 3, the following sequence of calculations would take place:

$$P_1 = \left(\frac{.15(16)}{16} \right) = .15,\ \text{for } 0 \le r \le 400 yds.$$
$$P_2 = \left(\frac{.3(8) + .15(8)}{16} \right) = .225,\ \text{for } 400 \le r \le 600 yds.$$
$$P_3 = \left(\frac{.6(4) + .3(4) + .15(8)}{16} \right) = .3,\ \text{for } 600 \le r \le 800 yds.$$
$$P_4 = \left(\frac{.3(8) + .15(8)}{16} \right) = .225,\ \text{for } 800 \le r \le 1000 yds.$$
$$P_5 = .15,\ \text{where } P_5 \text{ is the probability of kill beyond the } 4^{th} \text{ ring.}$$

Recall that EllipQ is a three parameter function that requires r, S_1 and S_2 as inputs. Where r is the radius of the applicable ring and both S_1 and S_2 are the semi-major and semi-minor axis of the target's 2-Sigma ellipse AOU. Let $S_1 = 1000$ yds and $S_2 = 500$ yds. The Pk algorithm would take on the following form and values:

$$Q_1(400,1000,500) = .8548$$

$$Q_2(600,1000,500) = .7085$$

$$Q_3(800,1000,500) = .5526$$

$$Q_4(1000,1000,500) = .4099$$

$$Pk = P_1(1-Q_1) + P_2(Q_1 - Q_2) + P_3(Q_2 - Q_3) + P_4(Q_3 - Q_4) + P_5 Q_4$$

$$Pk = .15(.1452) + .225(.1463) + .3(.1559) + .225(.1427) + .15(.4099)$$

$$Pk = .1951$$

All TEFF data is embedded, and S_1 and S_2 vary according to the EKF output. In addition, because all EKF calculations are performed in nautical miles, the input data where applicable is converted to nautical miles.

III. PACT-AKP PERFORMANCE TESTING

A. SCOPE

The correct kill probability calculation is premised on the proper tracking of the target. As referenced in Chapter II, the change in the state of the target is not only depicted in terms of its position and velocity but also described by its 2-Sigma AOU. Because the dimensions of the target relative to weapon water entry point 2-Sigma AOU semi-major and semi-minor axis are direct inputs for Washburn's EllipQ function, it is necessary to verify that such inputs are consistent with what the EKF is supposed to do.

The purpose of this chapter is to verify the proper functioning of PACT-AKP's EKF. A series of performance tests are conducted to test both the measurement and movement models as well as the proper graphing of the target's 2-Sigma AOU.

B. PACT-AKP MEASUREMENT MODEL PERFORMANCE TEST

1. Target Position Guess Reliability Test

The target's initial position guess will remain constant at Cardinal Coordinates (0, 0) as well as the area reference point at (35N/120E). As described in Chapter II, the initial hard-coded covariance matrix that describes the reliability of the "target position guess" is pre-established to be 5 nm. This means that if no measurements were taken and *t* remained at 0, the 2-sigma equiprobability contour describing the target state would be circular with a radius of 5nm.

The aforementioned statement holds true and is displayed in Figure 4.

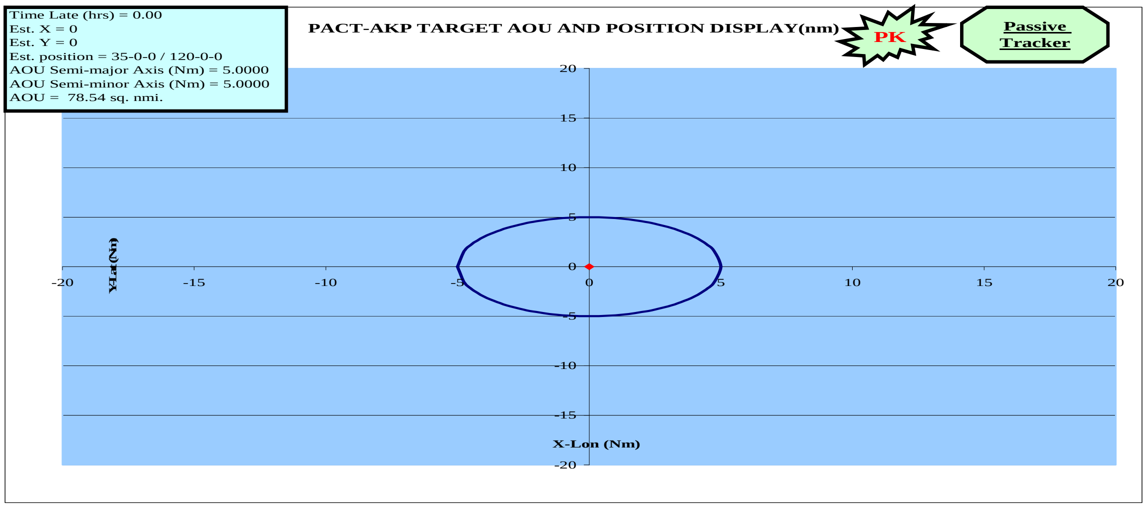


Figure 4. Target Position Guess AOU Display.

2. Single Sensor Measurement Model Performance Test

The purpose of this performance test is to exercise the measurement model of the EKF and verify the proper graphing of the 2-Sigma AOU. The target's initial position guess remains constant at Cardinal Coordinates (0, 0) as well as the area reference point at (35N/120E). The sensor standard deviation (σ) remains constant at 1.25 degrees.

a. PACT-AKP Measurement Model Single Sensor and Single Bearing-to-target AOU

(1) Case One: North-South Measurement

Measurement Station Position						Cartesian Coord				
Latitude			Longitude			X	Y	Brng-to-tgt	σ	Time
Deg	Min	Sec	Deg	Min	Sec			Deg	Deg	Hr
35	10	00	120	00	00	0	10	180	1.25	0

Table 1. North-South Single Sensor Bearing-to-target Measurement Station Data.

Based on the data from Table 1 and pre-established conditions for this performance test the following hypothesis is formulated:

Case One Hypothesis: If PACT-AKP's measurement model is processing the single bearing-to-target measurement correctly, the semi-major axis of the 2-Sigma Ellipse AOU is larger along the bearing-to-target axis. With the sensor σ being 1.25 degrees or .02 radians and the distance to the target at 10 nm, one would expect the semi-minor axis to be approximately 10 *(2*.02) = .436 nm.

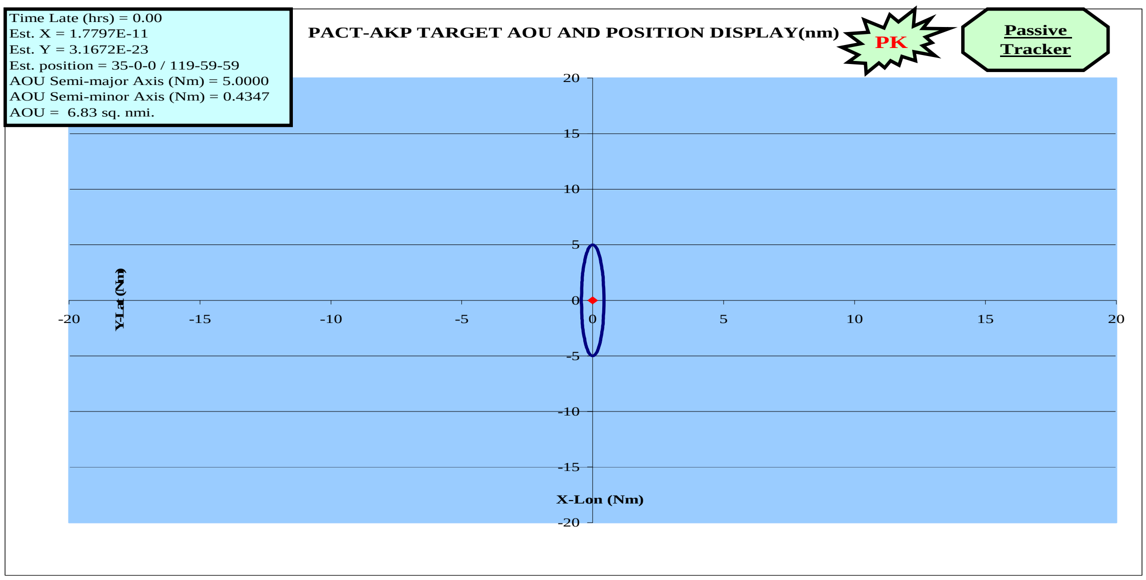


Figure 5. North-South Single Sensor Bearing-to-target Target AOU.

As observed from Figure 5 and consistent with data from Table 1, the measuring station is located at coordinate (0, 10) with a bearing-to-target measurement of 180 degrees. The resulting 2-sigma AOU has the semi-major axis oriented along the bearing-to-target axis (Y-axis) with a dimension of 4.9 nm and a semi-minor axis of length .435. The hypothesis is not rejected.

(2) Case Two: East-West Measurement. The same performance test and hypothesis as in case one applies to this case. This case uses measuring station data prescribed in Table 2.

Measurement Station Position						Cartesian Coord				
Latitude			Longitude			X	Y	Brng-to-tgt	σ	Time
Deg	Min	Sec	Deg	Min	Sec			Deg	Deg	Hr
35	00	00	119	47	45	10	0	270	1.25	0

Table 2. East-West Single Sensor Bearing-to-target Measurement Station Data.

The location of the measuring station is at (10, 0) and the bearing-to-target measurement is 270 degrees. The results are depicted in Figure 6.

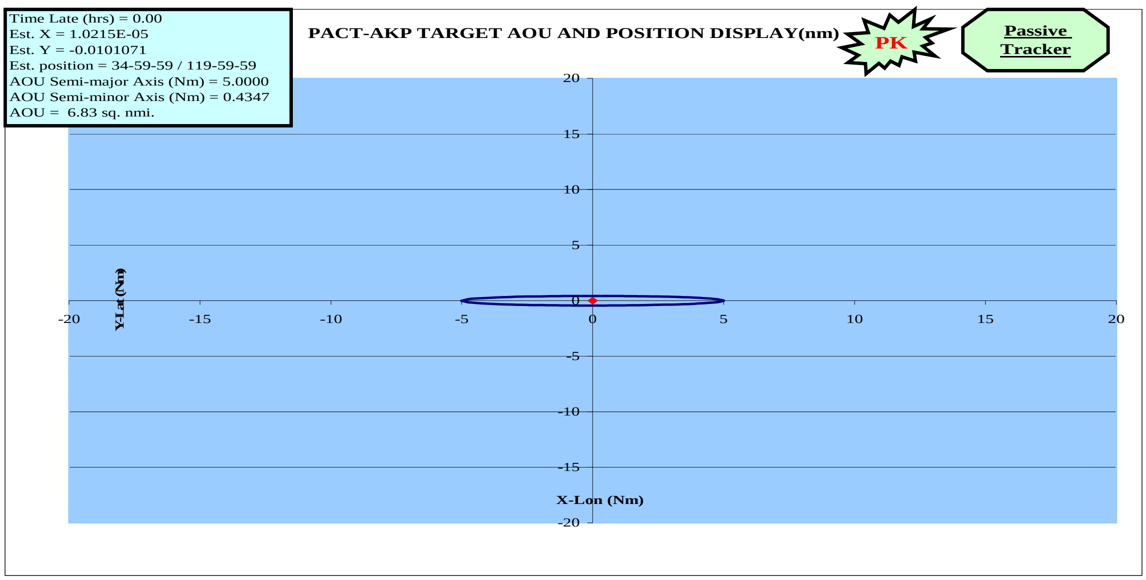


Figure 6. East-West Single Sensor Bearing-to-target Target AOU.

From Figure 6, the semi-major axis is oriented along the bearing-to-target axis (X-axis). Because the same conditions regarding the sensor sigma as in case one have not changed, the semi-minor axis' dimensions are proportionally equivalent. The hypothesis is not rejected.

b. PACT-AKP Measurement Model Single Bearing-to-target Position Test

Although the MTST measurement model performs all calculations on a flat earth it still accounts for the curvature of the earth when ascertaining the position of the target. If the measurement model is being implemented correctly this adjustment takes place at the Y axis coordinate of the target's position. Table 3 depicts the measuring station data used to conduct this performance test.

Measurement Station Position						Cartesian Coordinates				
Latitude			Longitude			X	Y	Brng-to-tgt	σ	Time
Deg	Min	Sec	Deg	Min	Sec			Deg	Deg	Hr
35	00	00	119	00	00	49.1	0	270	5	0

Table 3. Target Position Test Measuring Station Data.

After processing the data from Table 3, PACT-AKP estimates the position of the target to be located at (0, -.06). The negative coordinate in the Y axis is consistent with an adjustment for the earth's curvature. Based on these results one can not reject the hypothesis that the measurement model is functioning correctly.

3. Multiple Sensor Measurement Model Performance Test

The purpose of this test is to verify that PACT-AKP's measurement model is able to process more than one bearing-to-target measurement. The target's initial position guess will remain constant at Cardinal Coordinates (0, 0) as well as the latitude / longitude reference point at (35N/120E). The sensor standard deviation (σ) varies between 1.25 and 5 degrees.

a. Two Sensor Measurement Model Test

Table 4 depicts the measuring station data and Figure 7 the graphical display of the AOU. As it can be observed, the two measurements are orthogonal with the second measurement being less accurate than the first.

Station #	Measurement Station Position						Cartesian Coord				
	Latitude			Longitude					Brng-to-tgt	σ	Time
	Deg	Min	Sec	Deg	Min	Sec	X	Y	Deg	Deg	Hr
1	35	10	00	120	00	00	0	10	180	5	0
2	35	00	00	119	47	45	10	0	270	1.25	0

Table 4. Multiple Sensors Measuring Station Data.

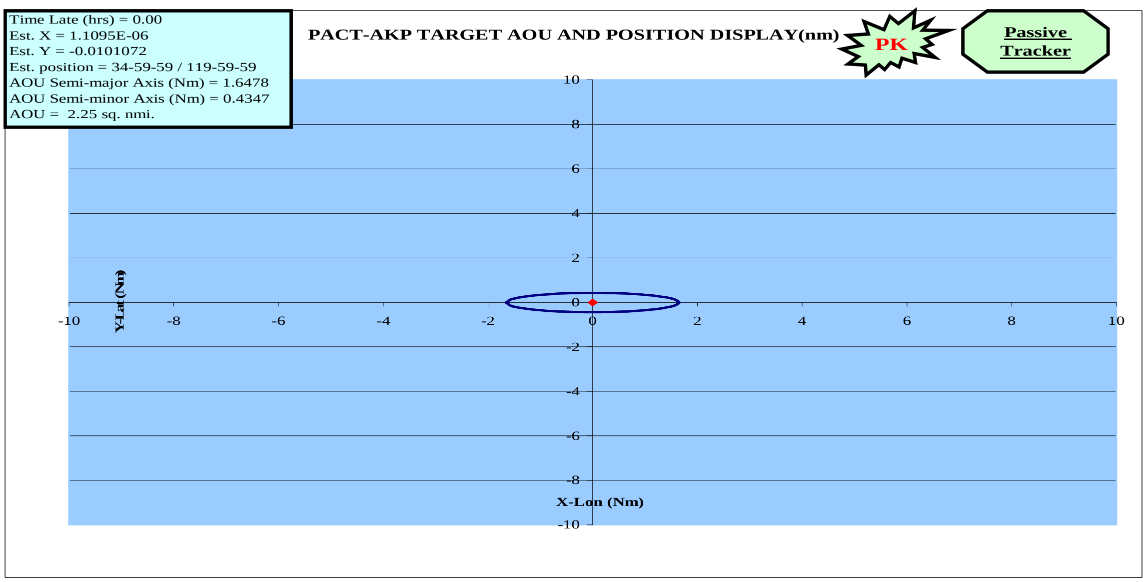


Figure 7. Two Sensor Measurement Target AOU.

From Figure 7, PACT-AKP estimates the position of the target to be at (0, -.01). In the case where no measurements were taken (Figure 4) the 2-sigma AOU had a radius of 5 nm, however, with two independent orthogonal measurements and different measurement accuracies the 2-sigma AOU becomes elliptical with a semi-major axis of

1.65 nm and semi-minor axis of .43 nm. The semi-major axis is oriented along the bearing-to-target measurement with the largest measurement error. On the other hand, if both bearing-to-target measurements shared the same sensor measurement error, the 2-sigma AOU would be circular with dimensions varying according to the sensor accuracy.

C. PACT-AKP MOVEMENT MODEL PERFORMANCE TEST

1. Change in Time Movement Model Test

The purpose of this performance test is to ensure that the dimensions of the target's AOU increase symmetrically with the passage of time given that no bearing-to-target measurements are taken. The target position guess is established at Cardinal Coordinates (0, 0) at start time 0. Table 5 depicts the results.

Time (Hr)	Semi-major Axis (Nm)	Semi-Minor Axis (Nm)
0	5	5
1	11.7	11.7
2	15.7	15.7
3	18.6	18.6
4	21.1	21.1

Table 5. Change in Time 2-Sigma AOU Expansion.

As observed from Table 5, the 2-sigma AOU expands symmetrically with time.

D. PACT-AKP EKF PERFORMANCE TEST

The purpose of this performance test is to exercise PACT-AKP's EKF to ensure that it's not only able to track a target, but also able to graph its corresponding AOU given multiple bearing-to-target measurements at different times with heterogeneous sensor bearing-to-target errors. This performance test is the most complex of all because

it involves simulating a moving target. For the purpose of this performance test, the generated target starts at (0, 0) and travel in a straight line from West to East at a speed of 7kts. Actual Target track information is depicted in Table 6. In terms of measurement station bearing-to-target information, to ensure that PACT-AKP is able to track the target, three bearing-to-target cases are explored. In all three cases, PACT-AKP processes bearing-to-target measurements that modify the "true" bearing-to-target measurement by adding an error that is normally distributed with mean 0 and standard deviation of 5 degrees. Measurement station data is depicted in Table 7.

1. Composite EKF Performance Test

True target location data is depicted in Table 6 and measurement station data is depicted in Table 7.

Time	Target Position						Cartesian Coordinates	
	Latitude			Longitude				
Hr	Deg	Min	Sec	Deg	Min	Sec	X	Y
0	35	00	00	120	00	00	0	0
1	35	00	00	119	51	25	7	0
2	35	00	00	119	42	50	14	0
3	35	00	00	119	34	15	21	0

Table 6. Actual Target Track Data.

As depicted in Table 6, the target travels on a straight line at a speed of 7kts. At time 0, the target is located at origin (0, 0), but after a change of three time periods (3 hrs) it travels 21 nm eastward.

Station #	Cartesian Coordinates of the Measurement Station							
			"Truth"	Case 1	Case 2	Case 3	σ	Time
	X	Y	Deg	Deg	Deg	Deg	Deg	Hr
1	0	10	180	180	185	182	5	0
2	10	0	270	271	268	275	5	0
1	7	10	180	178	176	182	5	1
2	17	-10	315	316	311	315	5	1
1	14	10	180	195	186	181	5	2
2	24	-10	315	315	316	317	5	2
1	21	10	180	180	181	172	5	3
2	31	0	270	270	276	265	5	3

Table 7. Measurement Station Data (Heterogeneous Sensor Error).

Table 7 depicts the measuring station positions at the time bearing-to-target measurements are taken. Because there is no passive signal generated by the target the positions of the measuring stations were determined so that plausible bearing-to-target measurements could be generated given the known time dependent target position. For contrast purposes the column labeled "truth" represents the "true" bearing-to-target measurement.

The end purpose of this test is to explore whether or not the various case scenario position estimate 2-sigma AOUs contain the true target. The true target position is depicted as a green diamond while the estimated target position is depicted as a smaller red diamond.

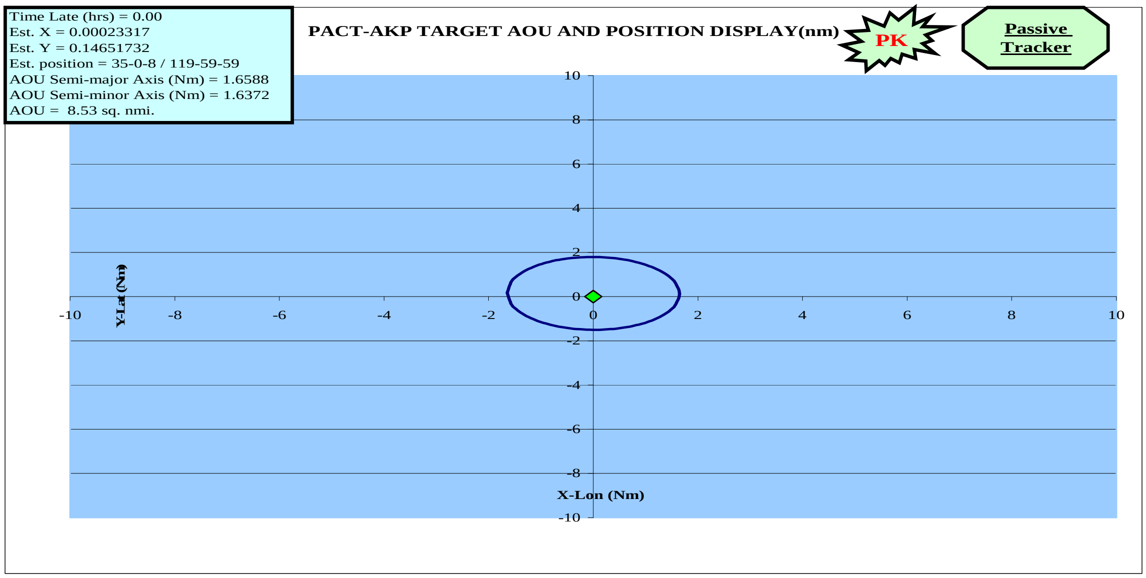


Figure 8. Case One Scenario at Time 0 After the First Two Measurements.

Figure 8 displays case one target position estimate after processing the first two bearing-to-target measurements. In this case, the 2-sigma AOU contains the true target position.

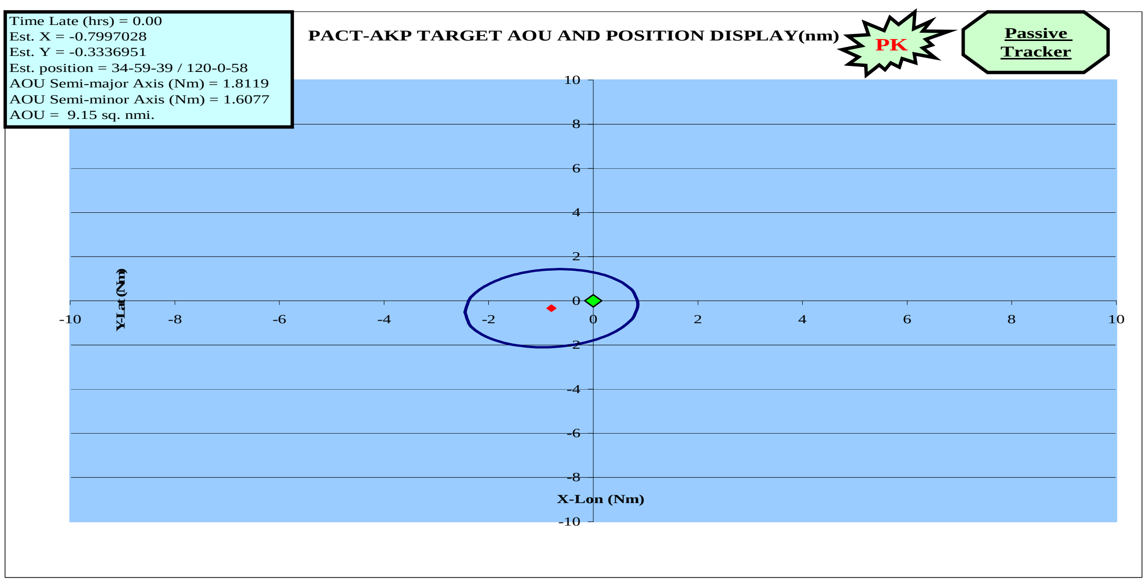


Figure 9. Case Two Scenario at Time 0 After the First Two Measurements.

Figure 9 depicts the 2-sigma AOU and estimated target position at time 0 for the case two scenario bearing-to-target measurements. In this case, the 2-sigma AOU contains the true target position.

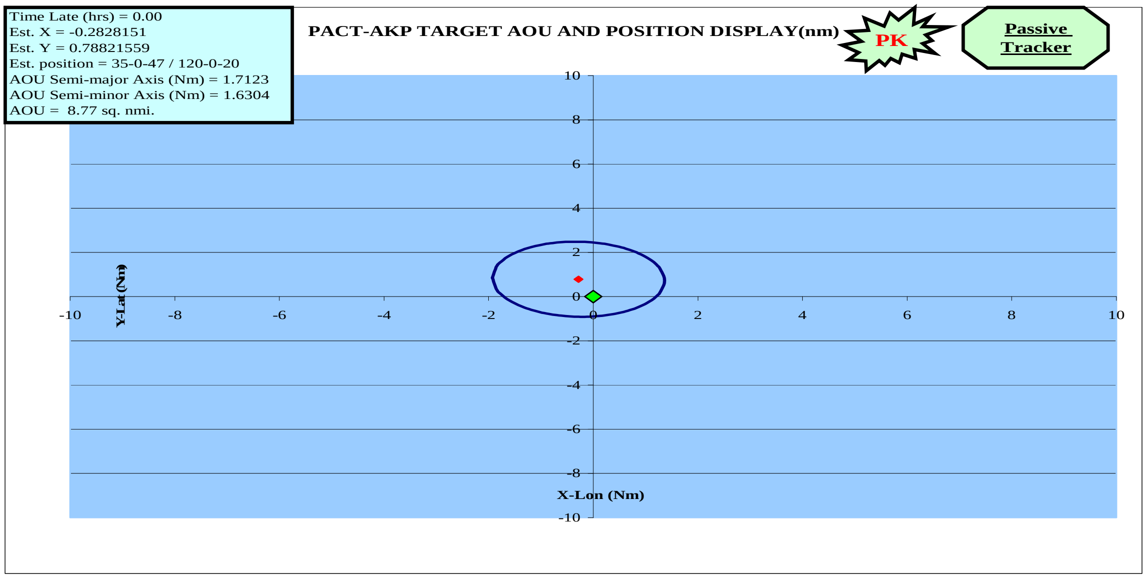


Figure 10. Case Three Scenario at Time 0 After the First Two Measurements.

Figure 10 displays PACT-AKP's target position and 2-sigma AOU estimate for case three scenario at time 0 after processing the first two bearing-to-target measurements. In this case, the 2-sigma AOU contains the true target position.

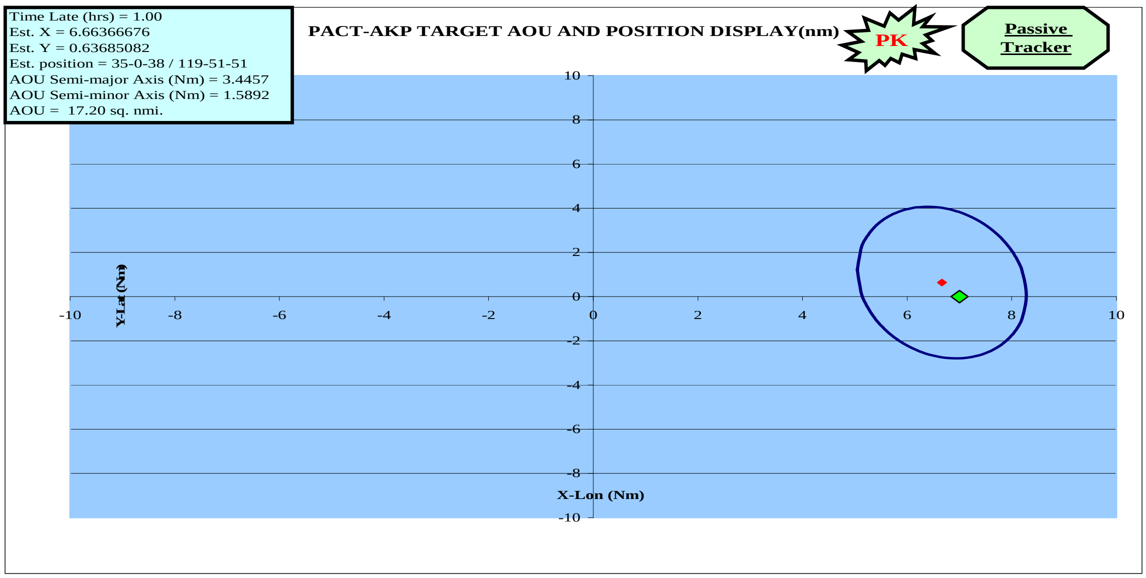


Figure 11. Case One Target Track at Time 1 After Four Bearing Measurements.

Figure 11 displays the case one estimated target position after processing four measurements. In this case, the 2-sigma AOU contains the true target position.

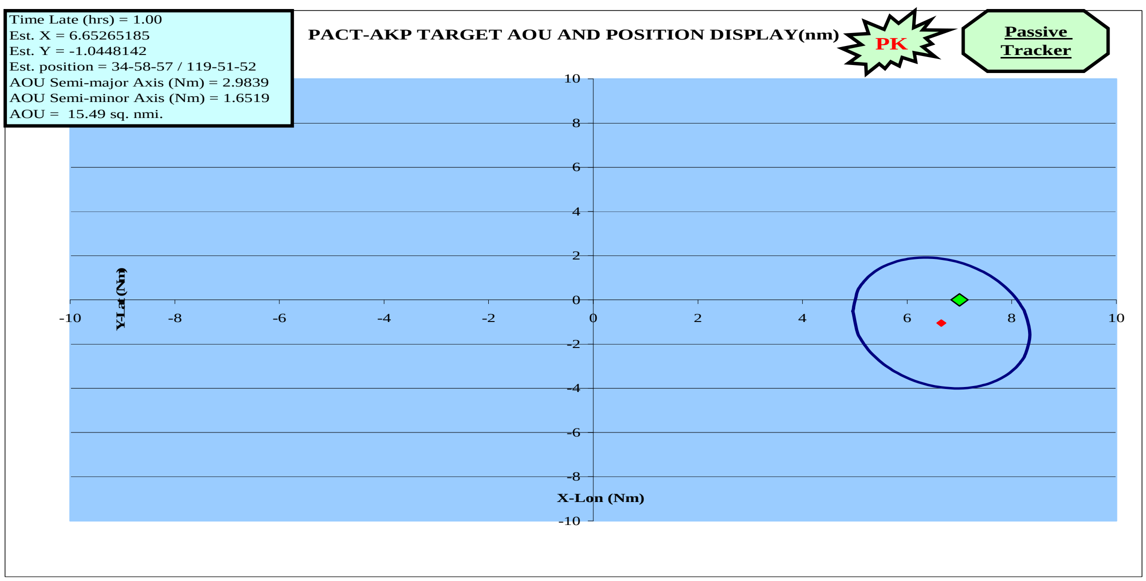


Figure 12. Case Two Scenario at Time 1 After Four Bearing Measurements.

Figure 12 depicts PACT-AKP's target position and 2-sigma AOU estimate for the bearing-to-target measurement for case two after four measurements. In this case the 2-sigma AOU contains the true target position.

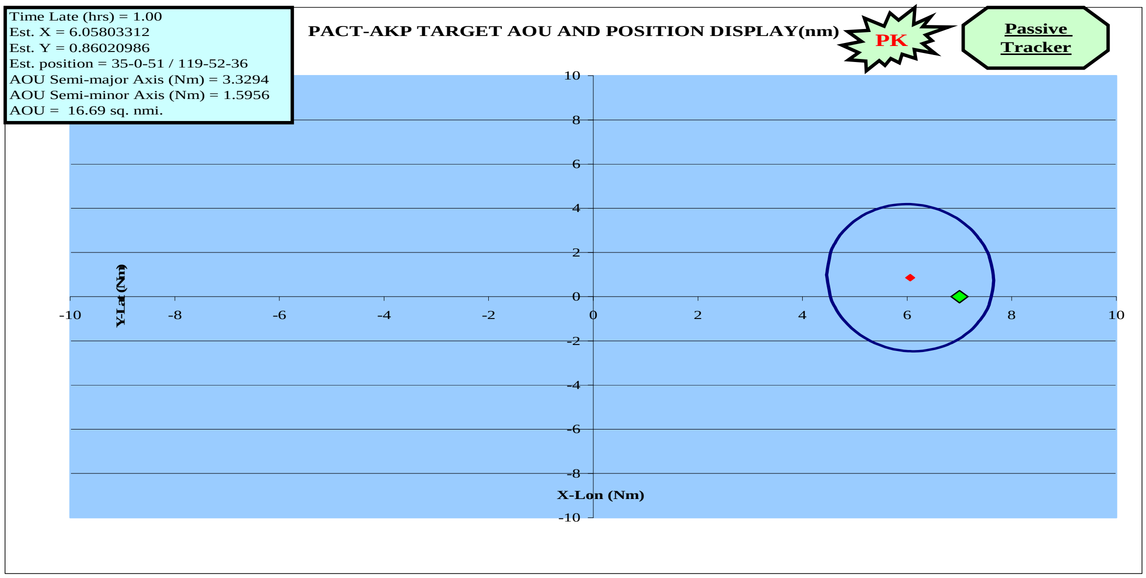


Figure 13. Case Three Scenario at Time 1 After Four Measurements.

Figure 13 displays the target's position and 2-sigma AOU estimation for the case three scenario at time 1. In this case the 2-sigma AOU contains the true target position.

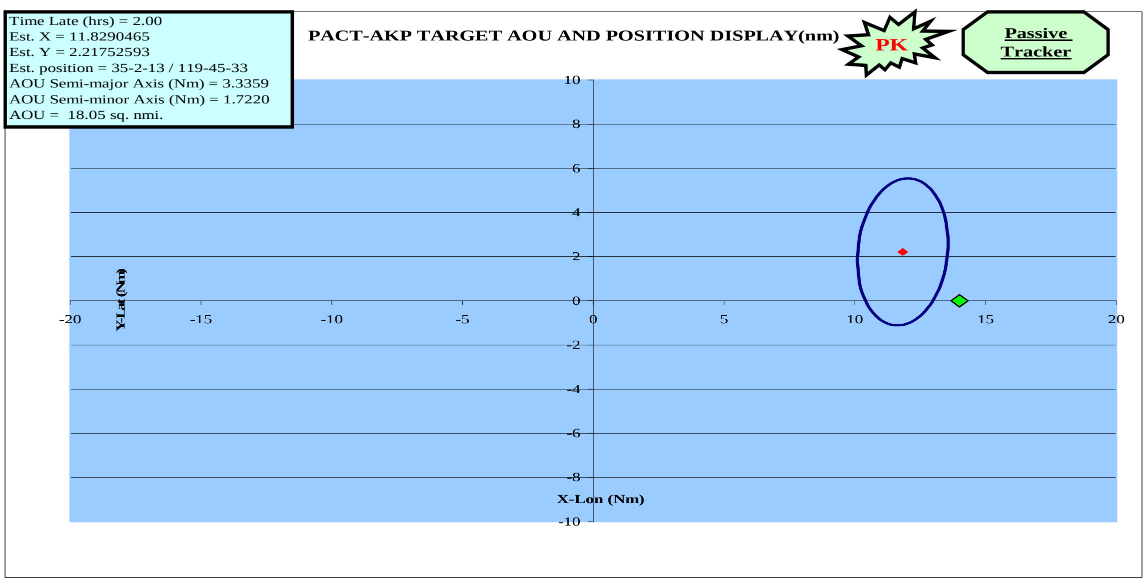


Figure 14. Case One Scenario at Time 2 After Six Measurements.

Figure 14 depicts PACT-AKP's target position estimation for case one at time two. As depicted, the target's true position is NOT contained by the 2-sigma AOU.

PACT-AKP processed a bearing-to-target measurement (195 degs) that we know is bad. At this point, PACT-AKP does not know any better since it has only processed six measurements. We can expect to notice a high dimensionless shock for one of the subsequent measurements.

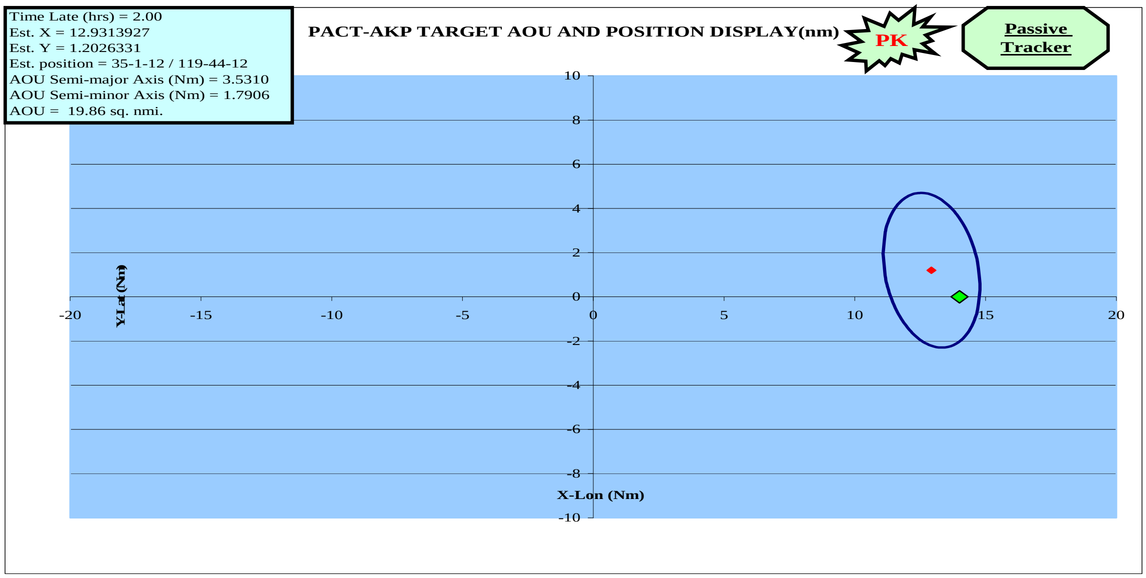


Figure 15. Case Two Scenario at Time 2 After Six Measurements.

Figure 15 depicts the case two scenario target position and 2-sigma AOU estimation after six bearing-to-target measurements. As observed, the 2-sigma AOU contains the target the true target's position.

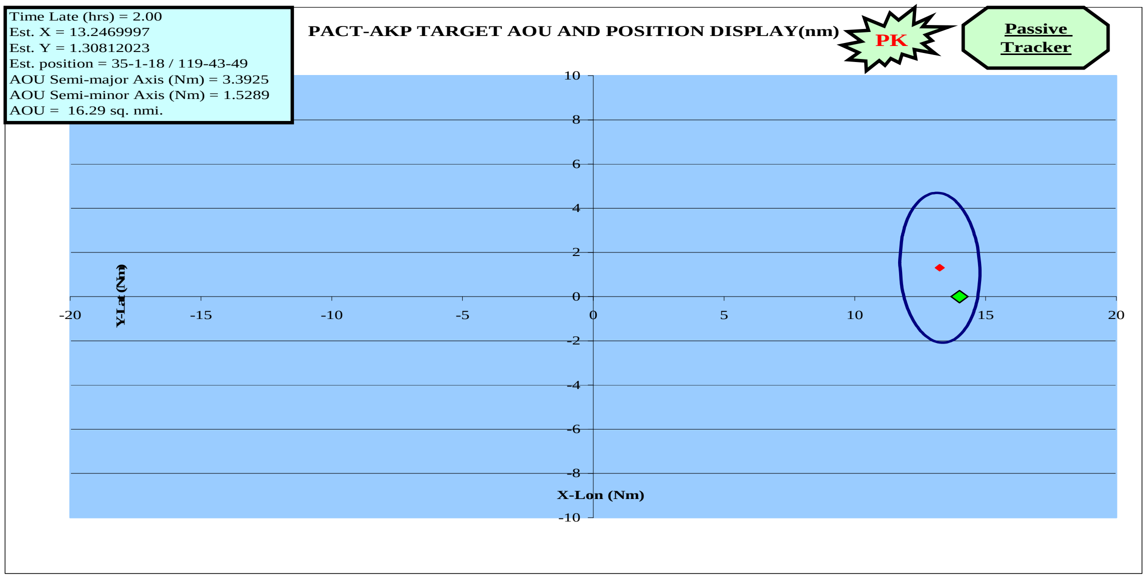


Figure 16. Case Three Scenario at Time 2 After Six Measurements.

Figure 16 depicts PACT-AKP's estimation of the target's position and 2-sigma AOU when processing the case three scenario bearing-to-target measurements. In this case the 2-sigma AOU contains the true target position.

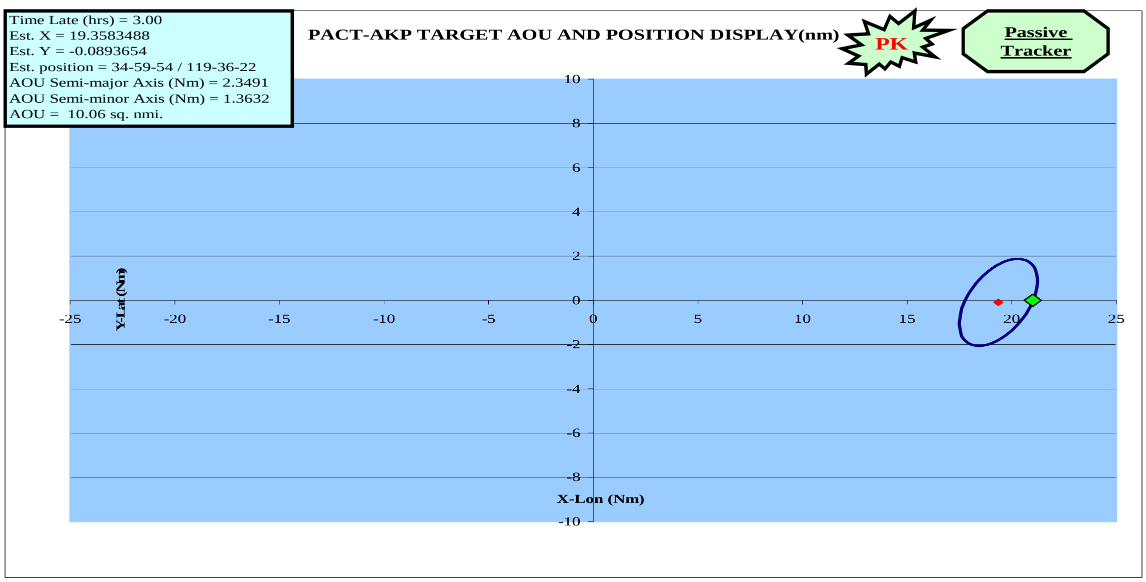


Figure 17. Case One Scenario at Time 3 After Six Measurements.

Figure 17 displays PACT-AKP's estimation of the target's position and 2-sigma AOU at time 3 for case one after six bearing measurements. The true target position is contained within the 2-sigma AOU.

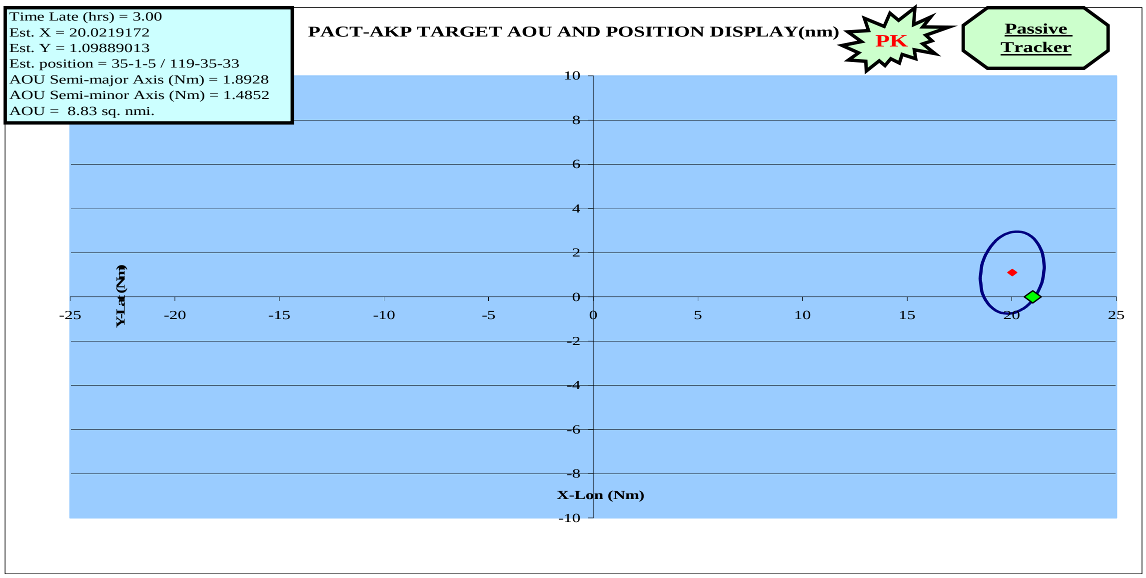


Figure 18. Case Two Scenario at Time 3 After Six Measurements.

Figure 18 depicts the target position and 2-sigma AOU estimation for case two scenario bearing-to-target measurements at time 3. In this case the 2-sigma AOU contains the true target position.

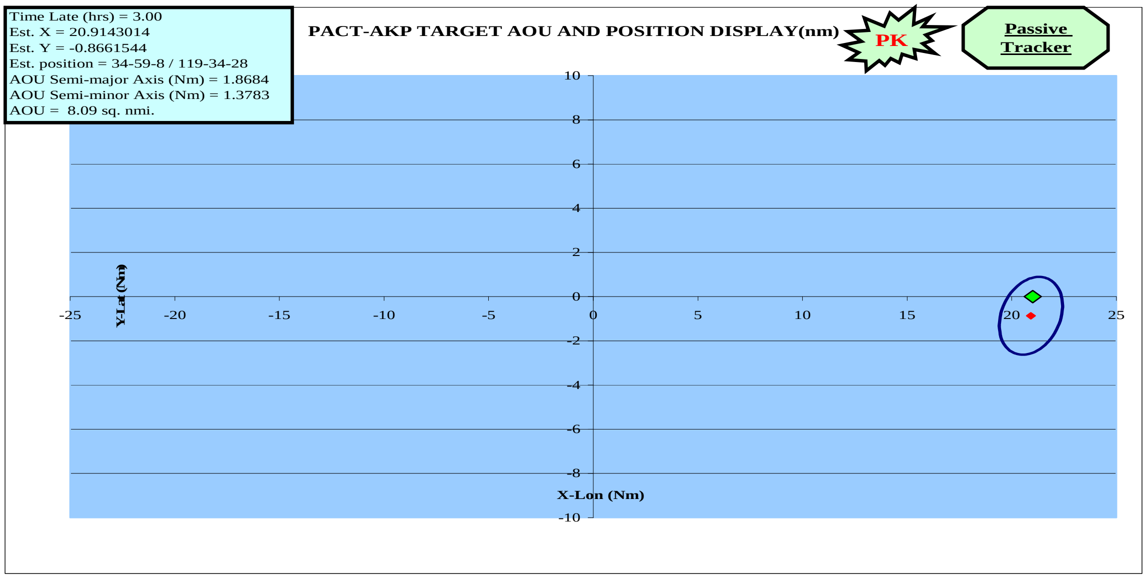


Figure 19. Case Three Scenario at Time 3 After Six Measurements.

Figure 19 shows PACT-AKP's target position and 2-sigma AOU estimation for case three scenario bearing-to-target measurements after six measurements. The generated 2-sigma AOU contains the target.

In 11 out of 12 (91.6%) scenario cases, the generated 2-sigma AOU contains the "true" target. This phenomenon can be attributed to the fact that the initial position guess and the target's initial position are the same. In addition, this test deals with a target that makes no course changes while PACT-AKP's MTST movement model expects two course changes per hour. In theory, the true position should be in the AOU 86.5% of the time.

2. Dimensionless Shock (DS) Test

Another effective measure of EKF performance is evaluating the average dimensionless shock (DS) for all case scenarios. The average DS is expected to be one if the EKF is tracking a target that is behaving in accordance with the MTST movement model. Recall that PACT-AKP assumes the target makes two course changes in a one hour period. For this performance test, the simulated target is assumed to travel in a straight line (no course changes) from West to East for a period of three hours. According to PACT-AKP's MTST movement model, PACT-AKP's EKF expects the target to

change course no less than six times. By assuming the target travels on a straight line, we are not only lying to the EKF but surprising enough we are doing the EKF a favor. Because of the simulated target's travel profile, one would expect the average DS to be less than one for all three scenarios. An average DS of less than one entails that PACT-AKP's EKF is having no trouble tracking a target. The measurement dimensionless shocks and their averages for all case scenarios are depicted in Table 8.

Measurement Dimensionless Shock			
Measurement #	Case One	Case Two	Case Three
1	0	.1	.02
2	0.004	.03	.11
3	1.51	1.99	1.37
4	0.08	.01	.06
5	0.7	1.27	2.37
6	0.52	.8	.17
7	3.31	2.14	2.65
8	0.04	.03	.06
AVERAGE	**0.77**	**.8**	**.85**

Table 8. Measurement Dimensionless Shocks.

Table 8 depicts the dimensionless shocks for all eight measurements processed in the sensor performance test. Dimensionless shock is an effective metric for ascertaining the accuracy of the bearing-to-target measurement in terms of what the EKF expects according to the conditions established in the MTST movement model. In essence, it's a good indication of whether or not the EKF is maintaining track of the target. When processing a measurement the goal is for the DS to be as close to zero as possible. As observed, the average DS for scenario one is .77, which indicates that the EKF is doing a better job at tracking the target than the other two scenarios despite its measurement seven having the largest DS. Recall, that measurement five was a bad bearing-to-target input, and when the operator inputs the correct bearings for the next measuring period it temporarily throws the EKF a "curve ball." PACT-AKP deemed measurement seven somewhat inaccurate as denoted by its DS. Scenarios two and three both have average dimensionless shocks of less than .9, which indicates that PACT-AKP's EKF is on average having little difficulty tracking the target.

E. PACT-AKP PROBABILITY OF KILL ALGORITHM TEST

The most unique feature of PACT-AKP is its kill probability function. As explained in Chapter II, the main inputs to the Pk algorithm from the EKF are the target relative to weapon water entry point AOU semi-major and semi-minor axis. Table 9 depicts the various “target relative to weapon water entry point” AOU dimensions for the target state estimations calculated in the composite EKF case two scenario heterogeneous sensor error performance test. The weapon’s CEP is assumed to be 400 yards and the TEFF data used in this test is as shown in Figure 3 and annotated in Table 10.

Target Relative to Weapon Water Entry Point AOU (Case Two Scenario)		
Measurement Time	Semi-major Axis (Nm)	Semi-minor Axis (Nm)
0	1.81	1.61
1	2.99	1.66
2	3.54	1.8
3	1.9	1.49

Table 9. Case Two Scenario Target Relative to Weapon Water Entry Point AOU Data.

r_n	Yards	Nautical Miles	P_n	
r_1	400	.1975	P_1	.15
r_2	600	.29624	P_2	.225
r_3	800	.39499	P_3	.3
r_4	1000	.49374	P_4	.225
			P_5	.15

Table 10. Converted TEFF Data.

Table 10 shows the weighted average of the TEFF scores per region. The weighted TEFF score averages along with their distances from weapon water entry point

are the building blocks of the Pk algorithm. The target relative to ASROC water entry point 2-sigma AOU (heterogeneous sensor error) case two scenario data is depicted in Table 11.

Measurement Time (t)	Semi-major Axis "S_1"(Nm)	Semi-minor Axis "S_2" (Nm)	AOU Area (Nm^2)
0	1.81	1.61	9.19
1	3	1.66	15.59
2	3.54	1.8	19.97
3	1.9	1.49	8.92

Table 11. Target relative to Torpedo Water Entry Point AOU Data.

Based on the data from Table 10 and Table 11, the resulting measurement time specific Pk results are depicted in Table 12.

Measurement Time (t)	Probability of Kill
0	.1534
1	.152
2	.1516
3	.1535

Table 12. Pk Results.

The probability of kill results as depicted in Table 12, show that the Pk against the submarine is at its highest at time 3 when the 2-sigma AOU area is at its smallest. These results show a relationship between the area of the 2-sigma AOU and Pk value highlighting the importance of not only the sensor accuracy, but also the frequency and number of bearing measurements.

In addition, suppose that $S_1 = S_2$ for values ranging from $S_1 = S_2 = 0$ to $S_1 = S_2 =$ .5 nm. Recall that the CEP of the weapon is 400 yards or .197 nm. The Pk results are depicted in Table 13 and Figure 20.

$S_1 = S_2$ (Nm)	Probability of Kill
0	.1976
.025	.1983
.05	.2003
.075	.2031
.1	.2059
.125	.2083
.15	.2097
.175	.2101
.197 (400 yards)	.2095
.225	.2077
.25	.2054
.275	.2028
.3	.1998
.325	.1968
.35	.1939
.375	.191
.4	.1882
.425	.1856
.45	.1832
.475	.181
.5	.1789

Table 13. Circular 2-Sigma AOU Probability of Kill Results.

Table 13 shows that the submarine's probability of kill is at its highest when $S_1 = S_2 = .175$ or 355 yards. The same holds true when $S_1 = S_2 = .165$ or 334 yards.

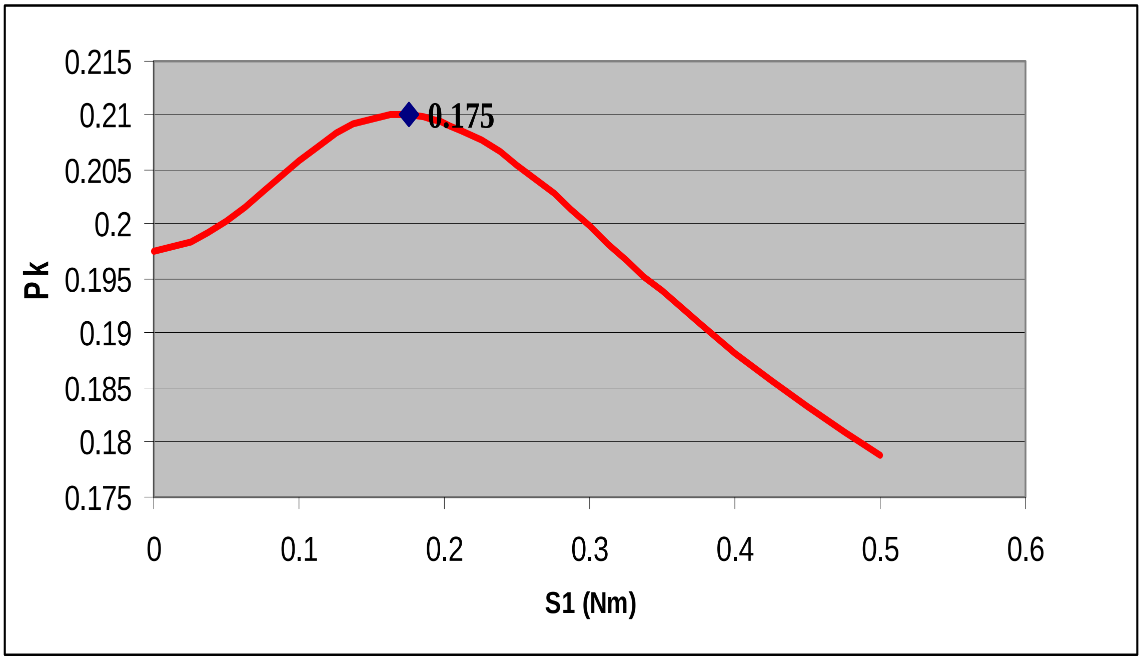


Figure 20. Pk vs. *S1*.

As observed from Figure 20, a small 2-sigma AOU may be in some cases detrimental because the ASROC will land too close to the origin, likewise, if the area of uncertainty is too large the value of the Pk will be low. In figure 20, a maximum Pk of .2101 is achieved when the 2-sigma area of uncertainty is between 173 and 195 yards^2. As grim as this result may be, one must keep in mind that torpedo TEFF data varies according to the type and evasion profile of the submarine as well as the type of torpedo and torpedo run settings. Depending on the torpedo effectiveness scores associated with the various rings the Pk will sometimes be higher and other times lower.

IV. CONCLUSION

A. INTRODUCTION

This thesis explores and answers a critical tactical decision aid requirement that is currently required by the surface Navy fleet; the probability of kill for VLA ASROC torpedo launch. PACT-AKP makes use of Kalman filter theory. Numerous other naval systems employ Kalman filters and variations of such, but what makes this thesis unique is the derivation of the probability of kill algorithm. Anti-submarine warfare operations are complex and beset with uncertainty. ASW operators have to not only contend with a dynamic battle space, but also with a myriad of other factors that often make anti-submarine warfare operations overwhelming exercises of educated guessing. The work developed in this thesis aids in bounding this uncertainty by tackling and solving the uncertainty surrounding the probability of kill for VLA ASROC torpedo launch. The passive contact tracker and probability of kill TDA developed in this thesis provides the commander with a tool capable of assessing the ASW deliberate attack engagement success, thus, providing him the knowledge necessary to better make decisions concerning limited shipboard ASW munitions expenditure.

B. RECOMMENDATIONS FOR FUTURE AREAS OF STUDY

PACT-AKP does not address optimal weapon load-out problems. The following ASW relevant area of future study is recommended:

1. Implement PACT-AKP calculated Pk to determine combatant commander area of responsibility (AOR) VLA ASROC inventory requirements through campaign modeling.
2. Explore optimal shipboard VLA ASROC inventory using Pk values and expected mission assignment.

3. Although PACT-AKP bases its computations of Pk on the assumption that the ASROC is aimed directly at the target, improvements may be possible by choosing other aimpoints.

C. CONCLUSIONS

PACT-AKP is an innovative and unique tactical decision aid that helps the commander make better informed decisions regarding the expenditure of limited ASW munitions. PACT-AKP enhances the TMA process by providing a mathematically sound platform from which the ASW Officer and commander alike can individually track a submarine based on passive bearing-to-target measurements, and compare the tracking results to the pencil and paper target motion analysis performed by the ASW team.

Overall, PACT-AKP not only addresses a tactically ASW relevant problem, but also in the process of doing so, it provides the commander with an alternative method of performing TMA and target kill probability knowledge before the expenditure of VLA ASROC munitions. The probability of kill algorithm developed herein is unique and will provide a credible foundation for the development of future Navy-wide ASW doctrine governing the use of ASROC munitions.

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